

HOSPITAL PERFORMANCE EVALUATION UNDER UNCERTAINTY USING NEUTROSOPHIC DEA: A COMPARISON OF CRS AND VRS MODELS FOR HEALTHCARE DEVELOPMENT

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Abstract: *In practice, finding public hospitals that are "on the frontier" continues to be a challenge for health authorities, and two issues are recurrent when measuring how efficiently hospitals spent the use of the budget: first, standard data envelopment analysis (DEA) is often not able to distinguish between "on the frontier" and "behind the frontier" hospitals; second, budget records are often incomplete and/or indeterminate. In this study, 32 hospitals in the public health sector in Tunisia are evaluated by neutrosophic DEA under both constant and variable returns to scale, and the neutrosophic CCR model (NCCR) and the neutrosophic BCC model (NBCC) are compared. The operating budgets are triangular neutrosophic numbers and are transformed into crisp ones based on a ranking function. After we account for scale differences the mean efficiency went from 0.676 to 0.763 under constant and variable returns respectively. By comparing the two perspectives with a scale-efficiency measure, the shortcomings of the two can be distinguished and health authorities can focus on two aspects: either managerial improvement or investing in capacity.*

Keywords: *Data envelopment analysis; Neutrosophic sets; Constant returns to scale; Variable returns to scale; Scale efficiency; Hospital efficiency*

Introduction

A significant proportion of national health spending goes to public hospitals, however their resources are largely fixed by central allocation. What a hospital can control is how much service it delivers from a given budget, and measuring that conversion is central to accountability and to sensible planning. The constant returns to scale (CRS) model of data envelopment analysis (DEA), introduced by Charnes, Cooper and Rhodes (1978), is the standard method for comparing such units and is now widely applied in healthcare (Kohl et al., 2019; Zubir et al., 2024). Under CRS, a small hospital is held to the same proportional standard as a large one. Banker, Charnes and Cooper (1984) later introduced the variable returns to scale (VRS) model, which compares each hospital only with peers of a similar size and so isolates managerial performance from the effect of operating at the wrong scale.

The obstacle to applying DEA to hospital records is that the models assume complete and exact figures, whereas hospital budgets are not always complete: some salary figures are unrecorded, and others are logged as zero for facilities that were clearly in operation. A linear programme cannot be run on such records unless the affected hospitals are dropped from the sample or their values are filled in, and either choice affects the outcome.

The neutrosophic approach offers a way past this difficulty. Drawing on the fuzzy set theory of Zadeh (1965) and the neutrosophic logic of Smarandache (1999), it represents each uncertain figure through three separate degrees: what is believed true, what is genuinely unknown, and what is thought false. The present study uses the same dataset of 32 regional hospitals in Tunisia examined by Abdelfattah (2021), who applied a triangular neutrosophic CCR model to handle the indeterminacy of the budget data. That study, however, kept a single CRS model and did not address the scale question.

This paper extends that starting point in two directions. First, a neutrosophic BCC (NBCC) model is developed and applied under variable returns to scale, so that managerial performance can be separated from scale effects within the same indeterminacy-tolerant framework. Second, the CRS and VRS results are combined through a scale-efficiency measure, which makes it possible to say whether a hospital's shortfall comes from how it is run or from the size at which it operates. Together, the two models give health authorities a sounder basis for deciding between managerial improvement and capacity investment.

Literature Review

DEA has a long record in health-services research. Since the seminal formulation of Charnes et al. (1978), the method has been used to benchmark hospitals, clinics and health centres across a wide range of settings, from district hospitals in Namibia (Zere et al., 2006) to public health centres in Kenya (Kirigia, Emrouznejad, Sambo, Munguti, & Liambila, 2004). The systematic review of Kohl et al. (2019) documents more than two hundred hospital DEA studies and notes that the choice between CRS and VRS specifications, and the selection of inputs and outputs, strongly shapes the conclusions drawn; the more recent review of input–output selection by Zubir et al. (2024) reaches a similar view. The two orientations are complementary rather than competing: the CRS model captures overall technical efficiency, while the VRS model isolates pure technical efficiency, and the ratio of the two gives scale efficiency, a diagnostic that tells a manager whether a shortfall reflects poor management or an unsuitable operating size (Cooper, Seiford, & Tone, 2007).

Handling uncertain or incomplete data within DEA has itself progressed through several stages. Fuzzy DEA, built on Zadeh (1965) and surveyed by Emrouznejad, Tavana, and Hatami-Marbini (2014), captures vagueness through membership functions, while interval and grey variants bound unknown values between limits. These methods describe how strongly a value belongs to a set, but they keep no separate account of what is simply unknown. Neutrosophic set theory, introduced by Smarandache (1999), supplies that missing account by attaching a distinct indeterminacy degree to every quantity. Edalatpanah (2018) set out how the representation fits within DEA, and a growing body of work has developed neutrosophic DEA models together with ranking and network extensions (Mohanta et al., 2023; Ayatollah et al., 2025; Mohanta et al., 2025). Applied to the Tunisian hospital data, Abdelfattah (2021) showed that indeterminate budgets can be accommodated without discarding any hospital.

One gap follows directly. Nearly all neutrosophic DEA applications adopt a single returns-to-scale specification, so the scale-efficiency diagnosis that requires both CRS and VRS scores has not been available under data indeterminacy. The present study addresses this gap by evaluating the same 32 regional hospitals in Tunisia under both the neutrosophic CCR (NCCR) and the neutrosophic BCC (NBCC) models, and by using the paired CRS and VRS scores to separate managerial from scale-related inefficiency.

Methodology

Data And Variables

This study draws on the 32 Tunisian regional hospitals reported by Abdelfattah (2021). Each hospital is treated as a decision-making unit (DMU), with a single input, the annual operating budget, set against three outputs: patient admissions, outpatient visits, and emergency visits. This one-input, three-output structure reflects the broader convention in hospital DEA research of weighing budgetary inputs against patient-volume outputs (Kirigia et al., 2004; Zere et al., 2006; Kohl et al., 2019), although the exact number of inputs and outputs varies across these studies. Since hospital managers have little say over how large their budget allocation is, but are nonetheless expected to turn it into as much patient care as possible, the models here are specified in the output-oriented direction. Table 1 summarises the variables used.

Table 1: Input And Output Variables

Type	Variable	Description
Input	Operating budget	Total annual operating expenditure, comprising government salary allocations, CNAM public-health-insurance contributions and net revenues, in Tunisian Dinars (TND).
Output 1	Patient admissions	Number of patients admitted for in-hospital care during the study year.
Output 2	Outpatient visits	Number of visits for diagnosis or treatment without overnight admission.
Output 3	Emergency visits	Number of cases handled by the emergency department.

Source: Abdelfattah (2021).

Eleven of the 32 hospitals have an indeterminate operating-budget figure. Menzel Bourguiba (DMU 4) is missing its salary figure; Tabarka (DMU 9) is missing both its salary figure and its CNAM contribution; and a further nine hospitals report a zero salary, which is implausible for

a facility that is actually operating. These eleven hospitals are treated as neutrosophic, as explained below, while the remaining 21 enter the analysis with their budgets as recorded.

Neutrosophic Data Representation

Each indeterminate budget is written as a triangular neutrosophic number (TNN), a lower, middle and upper estimate together with three membership degrees:

$$\tilde{x}_{ij} = \left\langle (x_{ij1}, x_{ij2}, x_{ij3}); t_{\tilde{x}_{ij}}, i_{\tilde{x}_{ij}}, f_{\tilde{x}_{ij}} \right\rangle \quad (1)$$

The truth degree t records the share of the budget that is directly observed, the indeterminacy degree i the part that cannot be verified, and the falsity degree f the part judged unreliable. Following Abdelfattah (2021), a hospital with two of three budget components recorded is given $t = 0.8, i = 0.3, f = 0.1$, while Tabarka, with only one component recorded, is given the lower truth degree $t = 0.7$. Each TNN is converted into a single crisp value through the ranking function

$$R(\tilde{x}_{ij}) = \frac{1}{4}(x_{ij1} + 2x_{ij2} + x_{ij3}) + (t_{\tilde{x}_{ij}} - i_{\tilde{x}_{ij}} - f_{\tilde{x}_{ij}}) \quad (2)$$

For the 21 fully recorded hospitals the recorded budget is used directly; the ranking function is applied only to the eleven indeterminate entries. All four models are solved on the resulting crisp dataset.

The Neutrosophic CCR Model

The neutrosophic CCR (NCCR) model of Abdelfattah (2021) embeds the ranking function in the output-oriented CCR multiplier formulation and measures overall technical efficiency under constant returns to scale. For the evaluated hospital,

$$\min \tilde{E}_0 = \sum_i v_i R(\tilde{x}_{i0}) \quad (3)$$

subject to:

$$\sum_r u_r y_{r0} = 1 \quad (4)$$

$$\sum_i v_i R(\tilde{x}_{ij}) - \sum_r u_r y_{rj} \geq 0, \quad j = 1, \dots, n \quad (5)$$

$$u_r \geq 0, \quad v_i \geq 0, \quad \forall r, i \quad (6)$$

where v_i and u_r are the input and output weights, Equations (3)–(6) are the multiplier (dual) form of the output-oriented DEA model: the weighted output of the evaluated hospital is normalised to one by Equation (4), while the objective minimises its weighted input. The optimal objective value therefore satisfies $\tilde{E}_0^* \geq 1$, and the efficiency score is reported as its reciprocal, $\text{Score} = 1 / \tilde{E}_0^*$, so that a hospital on the frontier scores one and a less efficient hospital scores below one.

The Neutrosophic BCC Model

The neutrosophic BCC (NBCC) model extends the VRS formulation of Banker et al. (1984) to indeterminate inputs. Variable returns to scale enter through a free scale variable c_0 , unrestricted in sign, which lets the frontier bend to register increasing, constant or decreasing returns:

$$\min \tilde{E}_0 = \sum_i v_i R(\tilde{x}_{i0}) + c_0 \quad (7)$$

subject to:

$$\sum_r u_r y_{r0} = 1 \quad (8)$$

$$\sum_i v_i R(\tilde{x}_{ij}) - \sum_r u_r y_{rj} + c_0 \geq 0, \quad j = 1, \dots, n \quad (9)$$

$$u_r \geq 0, \quad v_i \geq 0, \quad c_0 \text{ free in sign} \quad (10)$$

As in the NCCR model, Equations (7)–(10) are the multiplier form of the output-oriented programme, so the NBCC score is again the reciprocal of the optimal objective, $\text{Score} = 1 / \tilde{E}_0^*$. A hospital scoring one is pure-technically efficient for its scale, and the sign of c_0 at the optimum indicates whether it operates under increasing ($c_0 < 0$), constant ($c_0 = 0$) or decreasing ($c_0 > 0$) returns to scale.

Scale Efficiency

Pairing the CRS and VRS scores gives the neutrosophic scale efficiency,

$$\text{NSE}_j = \frac{\text{NCCR Score}_j}{\text{NBCC Score}_j} \quad (11)$$

which equals one when a hospital operates at its most productive scale and falls below one when part of its shortfall is a matter of scale rather than management. All programmes were solved in LINGO 21.0.

Results And Discussion

NCCR And NBCC Results

Table 2 reports the NCCR and NBCC results side by side. As can be seen in Table 2, under the NCCR model, three hospitals reach the constant-returns frontier: Khair-Eddine (DMU 2), M'Saken (DMU 15), and Houcine Bouzaïene (DMU 21). The mean score across the sample is 0.676, and the weakest performer is Kerkennah (DMU 18), at 0.209. These results are consistent with Abdelfattah (2021).

Table 2: NCCR And NBCC Efficiency Scores For The 32 Hospitals

DMU	Hospital	NCCR Score (Abdelfattah, 2021)	NBCC Score
1	Mahmoud El Matri	0.950	0.967
2	Khair-Eddine	1.000	1.000
3	Ben Arous	0.880	1.000
4	Menzel Bourguiba	0.414	0.655
5	Nabeul	0.868	0.946
6	Menzel Temime	0.552	0.558
7	Zaghouan	0.658	0.658
8	Jendouba	0.765	0.935
9	Tabarka	0.302	0.308
10	Beja	0.456	0.607
11	Medjez El Bab	0.594	0.670
12	M'hamed Bourguiba	0.535	0.725
13	Siliana	0.710	0.857
14	Kasserine	0.972	1.000
15	M'Saken	1.000	1.000
16	Moknine	0.538	0.560
17	Ksar Hellal	0.747	0.747
18	Kerkennah	0.209	0.250
19	Jebeniana	0.655	0.730
20	Mahres	0.726	0.896
21	Houcine Bouzaïene	1.000	1.000
22	Metlaoui	0.561	0.603
23	Tozeur	0.475	0.476
24	Sidi Bouzid	0.585	0.809
25	Ben Sassi de Gabès	0.687	1.000
26	Kebili	0.612	0.620
27	Habib Bourguiba	0.837	0.846
28	Sadok Mokadem	0.776	0.793
29	Zarzis	0.781	0.806
30	Ben Guerdanne	0.601	0.703
31	Tataouine	0.649	0.692
32	Nefta	0.528	1.000

Source: Authors' computation in LINGO 21.0. Bold scores denote hospitals efficient under NCCR and NBCC.

Under variable returns to scale, more hospitals make the frontier. Seven achieve pure technical efficiency under NBCC: Khair-Eddine (DMU 2), Ben Arous (DMU 3), Kasserine (DMU 14), M'Saken (DMU 15), Houcine Bouzaïene (DMU 21), Ben Sassi de Gabès (DMU 25), and Nefta (DMU 32). Four of these; DMU 3, 14, 25, and 32, are not efficient under NCCR, so their efficiency depends entirely on the VRS assumption: they run well for their size, but operate at a scale the CRS frontier would penalise. The mean NBCC score of 0.763 is well above the NCCR mean, which is expected, a frontier that follows the data more closely will always produce higher average scores.

Comparative Analysis and Scale Efficiency

Reading the constant-returns and variable-returns results together is the point of the exercise. Every hospital scores at least as well under NBCC as under NCCR, because the VRS frontier is the more forgiving benchmark, and the distance between the two scores measures the part of

a scale-efficiency ratio of roughly 0.53, over half its measured technical inefficiency under CRS traces to operating at too small a scale rather than to how it is run. Ben Sassi de Gabès (DMU 25, $NSE \approx 0.69$) and Ben Arous (DMU 3, $NSE \approx 0.88$) tell a version of the same story, in each case with the hospital reaching the VRS frontier while remaining short of the CRS one. Menzel Bourguiba (DMU 4) belongs in this group too, though it doesn't reach the VRS frontier either ($NBCC = 0.655$): its scale-efficiency ratio of roughly 0.63 is the second lowest in the entire sample, so even its more modest VRS shortfall is substantially a scale problem rather than a purely managerial one.

Between these two extremes sits a middle band that the figure makes visible but that a simple frontier count would miss. Fourteen of the 32 hospitals, including Sidi Bouzid, M'hamed Bourguiba, Beja, Mahres, Jendouba, Siliana, Kerkennah, and Ben Guerdenne, have a scale-efficiency ratio below 0.90, meaning at least a tenth of their combined shortfall is attributable to scale even though none of them reach the VRS frontier. A further thirteen sit between 0.90 and 1.00, close enough to the line that scale explains only a small residual of their shortfall. Only five hospitals sit exactly on the line at $NSE = 1.00$. In other words, most of the sample's inefficiency is predominantly managerial, but a substantial minority, roughly two in five hospitals, carries a scale component large enough to matter for policy, even where that hospital never reaches either frontier.

Tabarka (DMU 9) is the clearest illustration of the purely managerial case at the low end of performance. It sits close to the line, its scale-efficiency ratio of roughly 0.98 is among the highest in the sample, while its absolute scores (0.302 under NCCR, 0.308 under NBCC) are among the weakest. With almost no gap between the two views, there is no scale explanation available: whatever is driving Tabarka's poor performance is managerial, not a matter of operating at the wrong size. Kerkennah (DMU 18), which sits nearby in the lower-left of the plot with similarly weak absolute scores, is a different case: its scale-efficiency ratio of roughly 0.84 places it among the fourteen hospitals with a real scale component, so part of its weak performance should be read as a size problem layered on top of a managerial one, not a purely managerial shortfall.

Read this way, the figure does more than separate size problems from management problems for a handful of standout hospitals, it locates every hospital in the sample along that spectrum, from purely managerial cases at either end of the performance scale, through a substantial middle band with a real scale component, to the small group whose entire shortfall is a matter of size.

The count of efficient hospitals sums up the difference between the two views. The constant-returns model places three hospitals on the frontier, and the variable-returns model places seven, so NCCR is the stricter test of overall performance while NBCC isolates management from scale. Table 3 collects the headline figures for both models.

Table 3: Comparison of NCCR and NBCC Models

Model	Efficient DMUs	Efficient %	Mean Score	Note
NCCR	3	9.4	0.676	Overall technical efficiency (CRS)
NBCC	7	21.9	0.763	Pure technical efficiency (VRS)

Source: Authors' computation.

For a health authority, reading the two models together is more useful than relying on either alone. The variable-returns scores pick out hospitals that are well run for their size, and so serve as sensible benchmarks within their scale class, while the scale-efficiency figures flag hospitals whose main problem is size rather than management, a case for capacity adjustment rather than tighter operational control. The rankings should still be read with care: they reflect the chosen inputs, outputs, and assumptions, they don't measure quality of care, and they're best used to guide benchmarking rather than settle it.

Conclusion

This paper evaluated 32 Tunisian regional hospitals under two neutrosophic DEA models, NCCR and NBCC, so that the constant-returns and variable-returns views could be compared directly and joined through a scale-efficiency measure. Three hospitals reached the constant-returns frontier and seven reached the variable-returns frontier, with Khair-Eddine and Houcine Bouzaiene standing out as efficient under both. Setting the two views against each other separated hospitals whose shortfall is managerial from those whose shortfall is a matter of scale, a distinction that a single model cannot make.

For healthcare development, this distinction matters more than either score on its own. A hospital penalised mainly by scale is a candidate for capacity investment or network reorganisation, expanding it, merging its catchment with a neighbour, or feeding it more referrals, not for the tighter operational oversight that a purely managerial shortfall would call for. Treating the two as interchangeable risks pushing capacity fixes onto management problems, and vice versa, which wastes scarce reform effort. Read this way, the paired NCCR–NBCC analysis offers planners a screening tool for where to target investment against where to target reform, rather than a single ranking to act on uniformly across the system.

The study has limits that should be kept in view. It rests on a single cross-section of 32 hospitals from one year, the membership degrees are expert judgements, one ranking function is used, and the outputs are not adjusted for case mix, so a hospital treating harder cases may appear worse than it is. The scores are therefore best read as a first, scale-aware screen rather than a settled judgement. Later work could test the models on further datasets, vary the membership degrees and the ranking function, and extend the comparison to super-efficiency formulations or other discrimination-enhancing approaches that tighten the frontier further.

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