

A COMPARATIVE STUDY OF ADOMIAN DECOMPOSITION METHOD AND VARIATIONAL ITERATION METHOD IN SOLVING DAMPED BURGERS' EQUATION

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Article history

Received date : 10-9-2026

Revised date : 10-9-2026

Accepted date : 17-10-2026

Published date : 25-10-2026

To cite this document:

Wan Ramli, W. K. H., Mohd Khairul Anuar, M. A., Mohamad Rasat, N. A., Wan Yaacob, W. F., Md Nor, N., Zulkfli, S. N., Che Yusof, S. N. H., & Mohamad Sobri, N. (2026). A comparative study of Adomian decomposition method and variational iteration method in solving damped Burgers' equation. *Journal of Islamic, Social, Economics and Development (JISED)*, 11 (87), 937-950.

Abstract: Finding exact solutions to the nonlinear Burgers' equation remains challenging because of the interaction between nonlinear convection and linear diffusion. The main objective of this study is to evaluate and compare the accuracy, convergence, and algebraic simplicity of the Adomian Decomposition Method (ADM) and the Variational Iteration Method (VIM) when applied to damped Burgers' equations. Numerical computations are carried out in Maple up to the fifth iteration, while MATLAB is used to generate graphical plots for visual comparison. Results based on three different Gaussian-type initial profiles show that both ADM and VIM produce approximate solutions that closely agree to each other. Although both methods demonstrate comparable accuracy, VIM is computationally more efficient because it avoids the explicit construction of Adomian polynomials and has a shorter execution time using Maple.

Keywords: Adomian Decomposition, Burgers' Equation, Variational Iteration Method

Introduction

The study of nonlinear Partial Differential Equations (PDEs) is essential in applied mathematics because these equations describe fundamental real world processes, including fluid dynamics, wave propagation, and flow phenomena in complex physical systems. Among these, Burgers' equation is widely considered the simplest theoretical model that successfully incorporates nonlinear convection and diffusion processes (Tuz, 2023). It serves as a simplified framework for investigating the mathematical properties of nonlinear PDEs and understanding complex physical systems. Kasumo (2020) found that the equation is a major focus in shock theory and non-linear wave propagation because it has exact traveling shock wave solutions.

For this study, the analysis focuses on the general one-dimensional Burgers' equation, a quasi-linear parabolic PDE given by:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} \quad (1)$$

where $u(x, t)$ is the velocity field at position x and time t , $\frac{\partial u}{\partial t}$ represents temporal change,

$u \frac{\partial u}{\partial x}$ is the nonlinear convection term and $\nu \frac{\partial^2 u}{\partial x^2}$ is the diffusion term incorporating the kinematic viscosity coefficient ν . Mathematically, Eq. (1) captures the fundamental interplay between non-linear steepening and viscous dissipation. In addition to the standard model, modified formulations incorporating linear auxiliary terms are considered to account for linear damping effects (Bonkile et. al, 2018). The generalized model turns the hydrodynamic equation into a reaction-diffusion system:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} - \alpha u \quad (2)$$

In the present work, the auxiliary parameter is set to $\alpha = 1$ to investigate how semi-analytical methods handle problems with proportional decay. Altering the standard model with damping terms is a common practice when solving complex problems in acoustics and gas dynamics (Amir et al., 2023). Furthermore, this damped equation plays a significant role in modelling the evolution of flame fronts and viscous Marangoni flows, where damping directly affects solution stability and long-term evolution (Yan, 2023).

To address such non-linear complexities without relying on physical assumptions or restrictive linearization, semi-analytical techniques like the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM) are frequently utilized. Developed by Ji-Huan He in the late 1990s, VIM improves solutions iteratively using Lagrange multipliers. It has broad applications, including solving higher-order boundary-value problems (Noor & Mohyud-Din, 2008) and finding approximate solutions for the time-fractional Burgers' equation (Alwehebi et al., 2023).

Similarly, ADM is extensively applied to solve complex nonlinear differential, integral, and functional equations. As a powerful analytical tool, it does not require domain discretization, which makes it highly computationally efficient (Song et al., 2026). Demonstrating its versatility, ADM has been extended to highly complex nonlinear PDEs, yielding accurate and

explicit solutions for the fractional Burgers-Kuramoto KdV equation under the Hadamard approach (Hichem & Kacem, 2021).

Abu-Qouffa et al. (2021) and Abreu et al. (2022) compare these two methods in their research. Although both methods demonstrate strong computational efficiency, there is a distinct lack of literature addressing how variations in physical parameters affect the error propagation of ADM compared to VIM. Furthermore, a comprehensive comparative evaluation to identify which method maintains higher precision across the specified $0 < \nu < 1$ interval remains unexplored.

Adomian Decomposition Method (ADM)

ADM finds the solution by decomposing the equation into linear and non-linear components without relying on linearization or perturbation (Akpan, 2015; Alhendi & Alderremy, 2016). To implement the method, Eq. (2) is expressed in the following general operator form:

$$L[u] = R[u] - N[u] \quad (3)$$

where $L[u] = \frac{\partial u}{\partial t}$ is the time derivative, $N[u] = u \frac{\partial u}{\partial x}$ is the non-linear convection term, and

$R[u] = \nu \frac{\partial^2 u}{\partial x^2} - \alpha u$ is the diffusion operator. A critical component of this scheme is the inverse operator, L_t^{-1} which is defined as a definite integration with respect to time from 0 to t :

$$L_t^{-1} = \int_0^t dt$$

Applying this integration to the operator form will yield

$$u(x, t) = u(x, 0) + L_t^{-1} R[u] - L_t^{-1} N[u]$$

The nonlinear operator $N[u]$ can be decomposed by Adomian Polynomials A_n given by

$$N[u] = \sum_{n=0}^{\infty} A_n(x, t)$$

which the general form for A_n is expressed as:

$$A_n = u_0(u_n)_x + u_1(u_{n-1})_x + u_2(u_{n-2})_x + \dots + u_n(u_0)_x, \quad n = 0, 1, 2, \dots \quad (4)$$

Next, $u_n(x, t)$ will be determined recursively as follows:

$$\begin{aligned} u_0(x, t) &= u(x, 0) \\ u_{n+1}(x, t) &= L_t^{-1} \left(R \sum_{n=0}^{\infty} u_n \right) - L_t^{-1} \left(\sum_{n=0}^{\infty} A_n \right) \\ u_{n+1}(x, t) &= L_t^{-1} \left(\sum_{n=0}^{\infty} \nu u_n \right)_{xx} - L_t^{-1} \left(\sum_{n=0}^{\infty} \alpha u_n \right) - L_t^{-1} \left(\sum_{n=0}^{\infty} A_n \right) \end{aligned}$$

The solution is expressed as an infinite series:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + \dots$$

Variational Iteration Method (VIM)

VIM uses the correction functional to find the solution by successive approximations (Biazar & Aminikhah, 2009; Abdou & Soliman, 2005). The method is especially useful for solving non-linear problems since it provides a sequence of functions that converge rapidly to the exact solution. For the Burgers' equation, the correction functional is constructed by introducing a Lagrange multiplier, denoted by λ . The general expression for the functional is given by:

$$u_{n+1}(x, t) = u_n(x, t) + \int_0^t \lambda \left[\frac{\partial u_n}{\partial \tau} + u_n \frac{\partial u_n}{\partial x} + \alpha u_n - \nu \frac{\partial^2 u_n}{\partial x^2} \right] d\tau$$

By applying the stationary conditions of variational calculus, the multiplier is determined to be $\lambda = -1$. This optimal value is then substituted back into the functional to produce the following recursive iteration formula:

$$u_0(x, t) = u(x, 0)$$

$$u_{n+1}(x, t) = u_n(x, t) - \int_0^t \left[\frac{\partial u_n}{\partial \tau} + u_n \frac{\partial u_n}{\partial x} + \alpha u_n - \nu \frac{\partial^2 u_n}{\partial x^2} \right] d\tau$$

The solution of the problem is:

$$u = \lim_{n \rightarrow \infty} u_n$$

Numerical Applications

Problem 1

Consider the Burgers' Eq. (2) with $\alpha = 1$ and the initial condition:

$$u(x, 0) = e^{-x^2}$$

Adomian Decomposition Method

The Adomian polynomials A_n is constructed using the general formula for Adomian polynomials in Eq. (2) to the initial condition which gives:

$$A_0 = u_0 u_{0x} = -2xe^{-2x^2}$$

$$A_1 = u_0 u_{1x} + u_1 u_{0x} = \left[x^3 \left(-16e^{-2x^2} \right) + x^2 \left(-12e^{-3x^2} \right) + x \left(20e^{-2x^2} \right) + 2e^{-3x^2} \right] t$$

$$\vdots$$

and the decomposition terms as

$$\begin{aligned}
 u_0 &= e^{-x^2} \\
 u_1 &= L_t^{-1}(u_0)_{xx} - L_t^{-1}(u_0) - L_t^{-1}(A_0) \\
 &= \left[4x^2 e^{-x^2} + 2xe^{-2x^2} - 3e^{-x^2} \right] t \\
 u_2 &= L_t^{-1}(u_1)_{xx} - L_t^{-1}(u_1) - L_t^{-1}(A_1) \\
 &= \left[x^4 (8e^{-x^2}) + x^3 (24e^{-2x^2}) + x^2 (6e^{-3x^2} - 28e^{-x^2}) + x (-23e^{-2x^2}) + \left(\frac{17}{2} e^{-x^2} - e^{-3x^2} \right) \right] t^2 \\
 &\vdots
 \end{aligned}$$

Thus the ADM solution is the summation of

$$\begin{aligned}
 u &= u_0 + u_1 + u_2 + \dots \\
 &= e^{-x^2} + \left[4x^2 e^{-x^2} + 2xe^{-2x^2} - 3e^{-x^2} \right] t + \\
 &\quad \left[x^4 (8e^{-x^2}) + x^3 (24e^{-2x^2}) + x^2 (6e^{-3x^2} - 28e^{-x^2}) + x (-23e^{-2x^2}) + \left(\frac{17}{2} e^{-x^2} - e^{-3x^2} \right) \right] t^2 + \dots
 \end{aligned}$$

Variational Iteration Method

Starting with the initial condition $u(x, 0) = e^{-x^2}$ as $u_0(x, t)$, the next approximation is obtained using the recursive formula:

$$\begin{aligned}
 u_1(x, t) &= u_0(x, t) - \int_0^t \lambda \left[\frac{\partial u_0}{\partial \tau} + u_0 \frac{\partial u_0}{\partial x} + u_0 - \frac{\partial^2 u_0}{\partial x^2} \right] d\tau \\
 &= e^{-x^2} + \left[x^2 (4e^{-x^2}) + x (2e^{-2x^2}) - 2e^{-x^2} \right] t \\
 u_2(x, t) &= u_1(x, t) - \int_0^t \lambda \left[\frac{\partial u_1}{\partial \tau} + u_1 \frac{\partial u_1}{\partial x} + u_1 - \frac{\partial^2 u_1}{\partial x^2} \right] d\tau \\
 &= e^{-x^2} + \left[x^2 (4e^{-x^2}) + x (2e^{-2x^2}) - 2e^{-x^2} \right] t + \\
 &\quad \left[x^4 (8e^{-x^2}) + x^3 (24e^{-2x^2}) + x^2 (6e^{-3x^2} - 24e^{-x^2}) + x (-20e^{-2x^2}) + 6e^{-x^2} - e^{-3x^2} \right] t^2 + \\
 &\quad \left[x^5 (323e^{-2x^2}) + x^4 (16e^{-3x^2}) + x^3 (164e^{-4x^2} - 643e^{-2x^2}) + x^2 (-16e^{-3x^2}) + \right. \\
 &\quad \left. x (8e^{-2x^2} - 43e^{-3x^2}) + \frac{4}{3} e^{-3x^2} \right] t^3 \\
 &\vdots
 \end{aligned}$$

The subsequent iterations are calculated until $n = 5$ using Maple, as the calculations and output become increasingly lengthy.

Problem 2

Consider the Burgers' Eq. (2) with $\alpha = 1$ and the initial condition:

$$u(x, 0) = xe^{-x^2}$$

Adomian Decomposition Method

The Adomian polynomials A_n is constructed using the general formula for Adomian polynomials in Eq. (2) to the initial condition which gives

$$\begin{aligned} A_0 &= u_0 u_{0x} = (x - 2x^3)e^{-2x^2} \\ A_1 &= u_0 u_{1x} + u_1 u_{0x} = \left[x^5(-12e^{-3x^2} - 16e^{-2x^2}) + x^3(14e^{-3x^2} + 44e^{-2x^2}) + x(-2e^{-3x^2} - 14e^{-2x^2}) \right] t \\ &\vdots \end{aligned}$$

and the decomposition terms as

$$\begin{aligned} u_0 &= xe^{-x^2} \\ u_1 &= L_t^{-1}(u_0)_{xx} - L_t^{-1}(u_0) - L_t^{-1}(A_0) \\ &= \left[x^3(4e^{-x^2} + 2e^{-2x^2}) + x(-7e^{-x^2} - e^{-2x^2}) \right] t \\ u_2 &= L_t^{-1}(u_1)_{xx} - L_t^{-1}(u_1) - L_t^{-1}(A_1) \\ &= \left[x^5(6e^{-3x^2} + 24e^{-2x^2} + 8e^{-x^2}) + x^3(-7e^{-3x^2} - 59e^{-2x^2} - 44e^{-x^2}) + \right. \\ &\quad \left. x\left(e^{-3x^2} + \frac{39}{2}e^{-2x^2} + \frac{73}{2}e^{-x^2}\right) \right] t^2 \\ &\vdots \end{aligned}$$

Thus the ADM solution is the summation of

$$\begin{aligned} u &= u_0 + u_1 + u_2 + \dots \\ &= xe^{-x^2} + \left[x^3(4e^{-x^2} + 2e^{-2x^2}) + x(-7e^{-x^2} - e^{-2x^2}) \right] t + \\ &\quad \left[x^5(6e^{-3x^2} + 24e^{-2x^2} + 8e^{-x^2}) + x^3(-7e^{-3x^2} - 59e^{-2x^2} - 44e^{-x^2}) + \right. \\ &\quad \left. x\left(e^{-3x^2} + \frac{39}{2}e^{-2x^2} + \frac{73}{2}e^{-x^2}\right) \right] t^2 + \dots \end{aligned}$$

Variational Iteration Method

Starting with the initial condition $u(x, 0) = xe^{-x^2}$ as $u_0(x, t)$, the next approximation is obtained using the recursive formula.

$$\begin{aligned}
 u_1(x, t) &= u_0(x, t) - \int_0^t \lambda \left[\frac{\partial u_0}{\partial \tau} + u_0 \frac{\partial u_0}{\partial x} + u_0 - \frac{\partial^2 u_0}{\partial x^2} \right] d\tau \\
 &= x e^{-x^2} + \left[x^3 (2e^{-2x^2} + 4e^{-x^2}) + x(-e^{-2x^2}) - 6e^{-x^2} \right] t \\
 \\
 u_2(x, t) &= u_1(x, t) - \int_0^t \lambda \left[\frac{\partial u_1}{\partial \tau} + u_1 \frac{\partial u_1}{\partial x} + u_1 - \frac{\partial^2 u_1}{\partial x^2} \right] d\tau \\
 &= x e^{-x^2} + \left[x^3 (2e^{-2x^2} + 4e^{-x^2}) + x(-e^{-2x^2}) - 6e^{-x^2} \right] t + \\
 &\quad \left[x^5 (4e^{-2x^2} + 4e^{-x^2}) + x^3 (-16e^{-2x^2} - 20e^{-x^2}) + x(7e^{-2x^2} + 15e^{-x^2}) \right] t^2 + \\
 &\quad \left[x^7 \left(\frac{8}{3} e^{-2x^2} \right) + x^5 \left(-\frac{52}{3} e^{-2x^2} \right) + x^3 \left(\frac{82}{3} e^{-2x^2} - 643e^{-2x^2} \right) + x(-9e^{-2x^2}) \right] t^3 \\
 &\quad \vdots
 \end{aligned}$$

The subsequent iterations are calculated until $n = 5$ using Maple, as the calculations and output become increasingly lengthy.

Problem 3

Consider the Burgers' Eq. (2) with $\alpha = 1$ and the initial condition:

$$u(x, 0) = e^{-x^2} \sin x$$

Adomian Decomposition Method

The Adomian polynomials, A_n , is constructed using the general formula for Adomian polynomials in Eq. (2) to the initial condition which gives

$$\begin{aligned}
 A_0 &= u_0 u_{0x} = \left[-2x \sin^2(x) + \sin(x) \cos(x) \right] e^{-2x^2} \\
 A_1 &= u_0 u_{1x} + u_1 u_{0x} = \left[\begin{aligned} &-6e^{-2x^2} \sin(2x) + \frac{7}{4} e^{-3x^2} \sin(x) - \frac{5}{4} e^{-3x^2} \sin(3x) + \\ &x(12e^{-2x^2} - 16e^{-2x^2} \cos(2x) + 3e^{-3x^2} \cos(x) - 3e^{-3x^2} \cos(3x)) + \\ &x^2(12e^{-2x^2} \sin(2x) - 9e^{-3x^2} \sin(x) + 3e^{-3x^2} \sin(3x)) + \\ &x^3(8e^{-2x^2} \cos(2x) - 8e^{-2x^2}) \end{aligned} \right] t \\
 &\quad \vdots
 \end{aligned}$$

and the decomposition terms as

$$u_0 = e^{-x^2} \sin(x)$$

$$\begin{aligned}
 u_1 &= L_t^{-1}(u_0)_{xx} - L_t^{-1}(\alpha u_0) - L_t^{-1}(A_0) \\
 &= \left[\begin{aligned} &x^2 \left(4e^{-x^2} \sin(x) \right) + x \left(e^{-x^2} - e^{-2x^2} \cos(2x) - 4e^{-2x^2} \cos(x) \right) \\ &-\frac{1}{2} e^{-2x^2} \sin(2x) - 4e^{-x^2} \sin(x) \end{aligned} \right] t \\
 u_2 &= L_t^{-1}(u_1)_{xx} - L_t^{-1}(\alpha u_1) - L_t^{-1}(A_1) \\
 &= \left[\begin{aligned} &16e^{-x^2} \sin(x) + \frac{29}{4} e^{-2x^2} \sin(2x) - \frac{7}{8} e^{-3x^2} \sin(x) + \frac{5}{8} e^{-3x^2} \sin(3x) \\ &x \left(32e^{-x^2} \cos(x) - \frac{25}{2} e^{-2x^2} + \frac{41}{2} e^{-2x^2} \cos(2x) - \frac{3}{2} e^{-3x^2} \cos(x) + \frac{3}{2} e^{-3x^2} \cos(3x) \right) + \\ &x^2 \left(\frac{9}{2} e^{-3x^2} \sin(x) - \frac{3}{2} e^{-3x^2} \sin(3x) - 18e^{-2x^2} \sin(2x) - 40e^{-x^2} \sin(x) \right) + \\ &x^3 \left(12e^{-2x^2} - 12e^{-2x^2} \cos(2x) - 16e^{-x^2} \cos(x) \right) + x^4 \left(8e^{-x^2} \sin(x) \right) \end{aligned} \right] t^2 \\
 &\vdots
 \end{aligned}$$

Thus the ADM solution is the summation of

$$\begin{aligned}
 u &= u_0 + u_1 + u_2 + \dots \\
 &= e^{-x^2} \sin(x) + \left[\begin{aligned} &x^2 \left(4e^{-x^2} \sin(x) \right) + x \left(e^{-x^2} - e^{-2x^2} \cos(2x) - 4e^{-2x^2} \cos(x) \right) \\ &-\frac{1}{2} e^{-2x^2} \sin(2x) - 4e^{-x^2} \sin(x) \end{aligned} \right] t + \\
 &\left[\begin{aligned} &16e^{-x^2} \sin(x) + \frac{29}{4} e^{-2x^2} \sin(2x) - \frac{7}{8} e^{-3x^2} \sin(x) + \frac{5}{8} e^{-3x^2} \sin(3x) \\ &x \left(32e^{-x^2} \cos(x) - \frac{25}{2} e^{-2x^2} + \frac{41}{2} e^{-2x^2} \cos(2x) - \frac{3}{2} e^{-3x^2} \cos(x) + \frac{3}{2} e^{-3x^2} \cos(3x) \right) + \\ &x^2 \left(\frac{9}{2} e^{-3x^2} \sin(x) - \frac{3}{2} e^{-3x^2} \sin(3x) - 18e^{-2x^2} \sin(2x) - 40e^{-x^2} \sin(x) \right) + \\ &x^3 \left(12e^{-2x^2} - 12e^{-2x^2} \cos(2x) - 16e^{-x^2} \cos(x) \right) + x^4 \left(8e^{-x^2} \sin(x) \right) \end{aligned} \right] t^2 + \dots
 \end{aligned}$$

Variational Iteration Method

Starting with the initial condition $u(x, 0) = e^{-x^2} \sin(x)$ as $u_0(x, t)$, the next approximation is obtained using the recursive formula.

$$\begin{aligned}
 u_1(x, t) &= u_0(x, t) - \int_0^t \lambda \left[\frac{\partial u_0}{\partial \tau} + u_0 \frac{\partial u_0}{\partial x} + u_0 - \frac{\partial^2 u_0}{\partial x^2} \right] d\tau \\
 &= e^{-x^2} \sin(x) + \left[\begin{aligned} &x^2 \left(4e^{-x^2} \sin(x) \right) + x \left(2e^{-2x^2} \sin^2(x) - 4e^{-x^2} \cos(x) \right) + \\ &\left(-e^{-2x^2} \sin(x) \cos(x) - 3e^{-x^2} \sin(x) \right) \end{aligned} \right] t \\
 &\vdots
 \end{aligned}$$

The subsequent iterations are calculated until $n = 5$ using Maple, as the calculations and output become increasingly lengthy.

Solution Analysis

In this study, each problem is evaluated at the spatial domain restricted to $x \in [0.2, 2.0]$.

Table 1: Approximate Value (ADM and VIM) for Problem 1

x	ADM for $\nu = 1$	VIM for $\nu = 1$	ADM for $\nu = 0.1$	VIM for $\nu = 0.1$	ADM for $\nu = 0.01$	VIM for $\nu = 0.01$
0.2	0.9374485411	0.9374485410	0.9530205589	0.9530205588	0.9546213115	0.9546213112
0.4	0.8376688755	0.8376688755	0.8481924102	0.8481924104	0.8492647798	0.8492647799
0.6	0.6922336837	0.6922336839	0.6961057201	0.6961057204	0.6964849436	0.6964849438
0.8	0.5289771167	0.5289771168	0.5267478638	0.5267478639	0.5264974612	0.5264974612
1.0	0.3738834627	0.3738834631	0.3676400623	0.3676400620	0.3669833793	0.3669833794
1.2	0.2445420838	0.2445420834	0.2367964005	0.2367964006	0.2359966620	0.2359966617
1.4	0.1480795954	0.1480795953	0.1408275523	0.1408275524	0.1400892795	0.1400892794
1.6	0.0830441822	0.0830441822	0.0773589443	0.0773589444	0.0767880408	0.0767880408
1.8	0.0431386148	0.0431386147	0.0392557368	0.0392557368	0.0388713711	0.0388713711
2.0	0.0207574096	0.0207574097	0.0184016920	0.0184016920	0.0181720719	0.0181720720

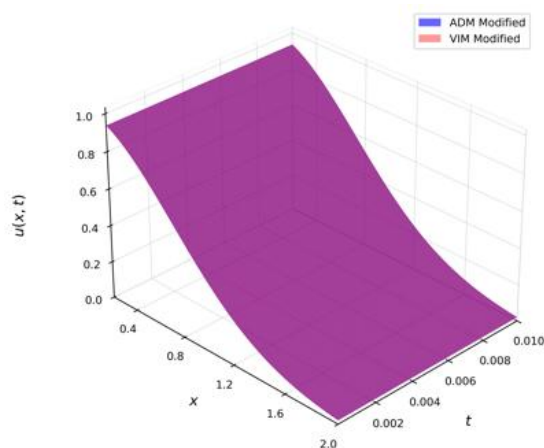
Table 2: Approximate Value (ADM and VIM) for Problem 2

x	ADM for $\nu = 1$	VIM for $\nu = 1$	ADM for $\nu = 0.1$	VIM for $\nu = 0.1$	ADM for $\nu = 0.01$	VIM for $\nu = 0.01$
0.2	0.1782459331	0.1782459330	0.1875075962	0.1875075963	0.1884770267	0.1884770266
0.4	0.3184577632	0.3184577634	0.3337477305	0.3337477304	0.3353431026	0.3353431026
0.6	0.3954268660	0.3954268659	0.4117405303	0.4117405303	0.4134316797	0.4134316798
0.8	0.4040370180	0.4040370180	0.4167876787	0.4167876783	0.4180929370	0.4180929367
1.0	0.3580325147	0.3580325147	0.3648042835	0.3648042833	0.3654773919	0.3654773917
1.2	0.2816809734	0.2816809733	0.2826578329	0.2826578333	0.2827295110	0.2827295113
1.4	0.1992967093	0.1992967095	0.1963984510	0.1963984510	0.1960785109	0.1960785108
1.6	0.1278238171	0.1278238171	0.1233663444	0.1233663445	0.1228987791	0.1228987789
1.8	0.0747123094	0.0747123094	0.0704301416	0.0704301416	0.0699918640	0.0699918640
2.0	0.0399402943	0.0399402943	0.0366769162	0.0366769162	0.0363495237	0.0363495237

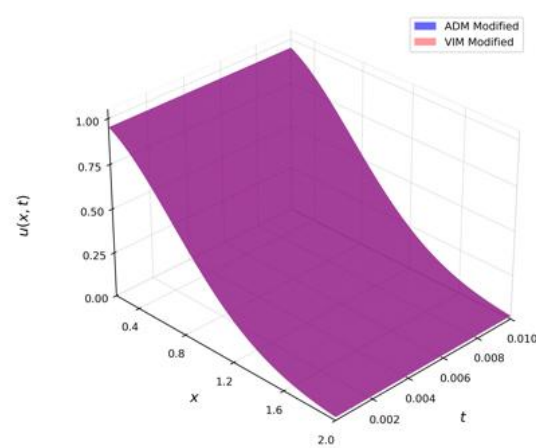
Table 3: Approximate Value (ADM and VIM) for Problem 3

x	ADM for $\nu = 1$	VIM for $\nu = 1$	ADM for $\nu = 0.1$	VIM for $\nu = 0.1$	ADM for $\nu = 0.01$	VIM for $\nu = 0.01$
0.2	0.1755050055	0.1755050055	0.1861208323	0.1861208323	0.1872396928	0.1872396924
0.4	0.3078692064	0.3078692064	0.3248565323	0.3248565323	0.3266394342	0.3266394335
0.6	0.3705746101	0.3705746102	0.3876310732	0.3876310732	0.3894051990	0.3894051993
0.8	0.3621426296	0.3621426296	0.3740239271	0.3740239271	0.3752365592	0.3752365594
1.0	0.3025840672	0.3025840673	0.3072649951	0.3072649951	0.3077148784	0.3077148780
1.2	0.2210838575	0.2210838574	0.2197864960	0.2197864960	0.2196186905	0.2196186907
1.4	0.1428973955	0.1428973955	0.1384599945	0.1384599945	0.1379822580	0.1379822582
1.6	0.0821992425	0.0821992425	0.0772627387	0.0772627387	0.0767506062	0.0767506062
1.8	0.0421817977	0.0421817977	0.0382559447	0.0382559447	0.0378593208	0.0378593207
2.0	0.019296866	0.0192967866	0.0167765852	0.0167765852	0.0165283731	0.0165283731

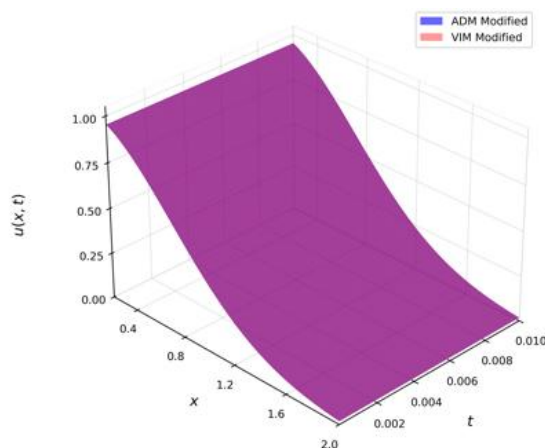
Tables 1, 2 and 3 show the approximate solution of Burgers' equation using ADM and VIM at $t = 0.01$ for various values of ν . These values show that there is no discernible difference between the numerical performances of ADM and VIM when the damping term is incorporated into Burgers' equation. This is because both techniques can handle this linear term stably and consistently, producing results that are essentially equivalent at all points. The effect of the damping term, which is intended to alter the characteristics of the solution, is handled equivalently by both techniques, and no increase in discrepancy is observed. Thus, the high degree of similarity between ADM and VIM outputs demonstrates inter-method agreement or solution consistency.



(a) $\nu = 1$

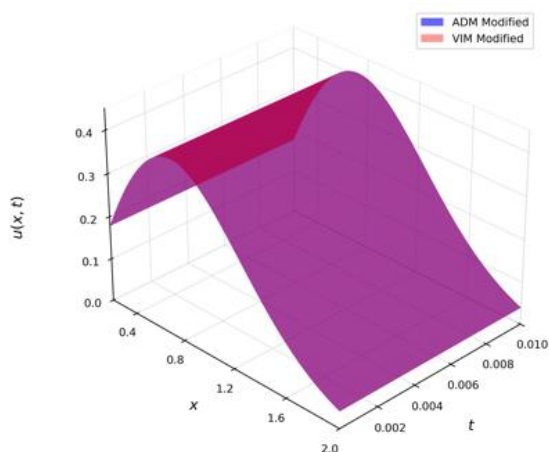


(b) $\nu = 0.1$

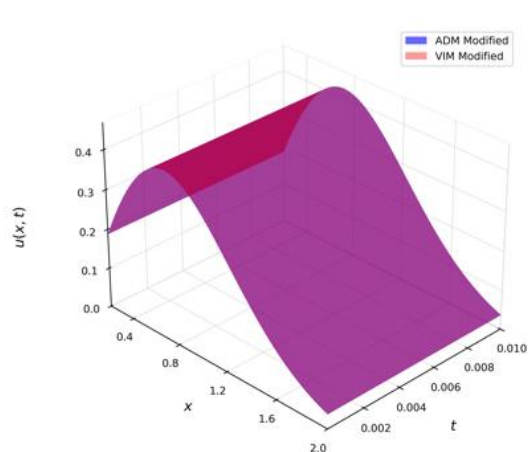


(c) $\nu = 0.01$

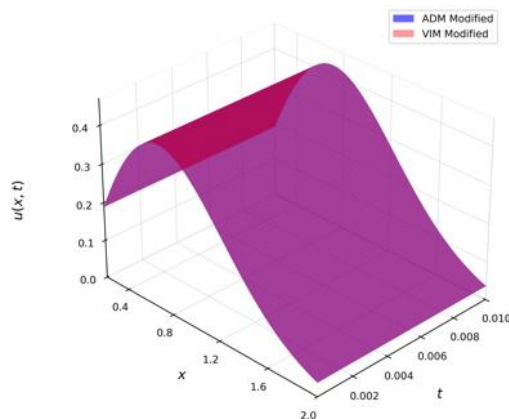
Figure 1: Solution curves of ADM and VIM for different values of ν for Problem 1



(a) $\nu = 1$



(b) $\nu = 0.1$



(c) $\nu = 0.01$

Figure 2: Solution curves of ADM and VIM for different values of ν for Problem 2

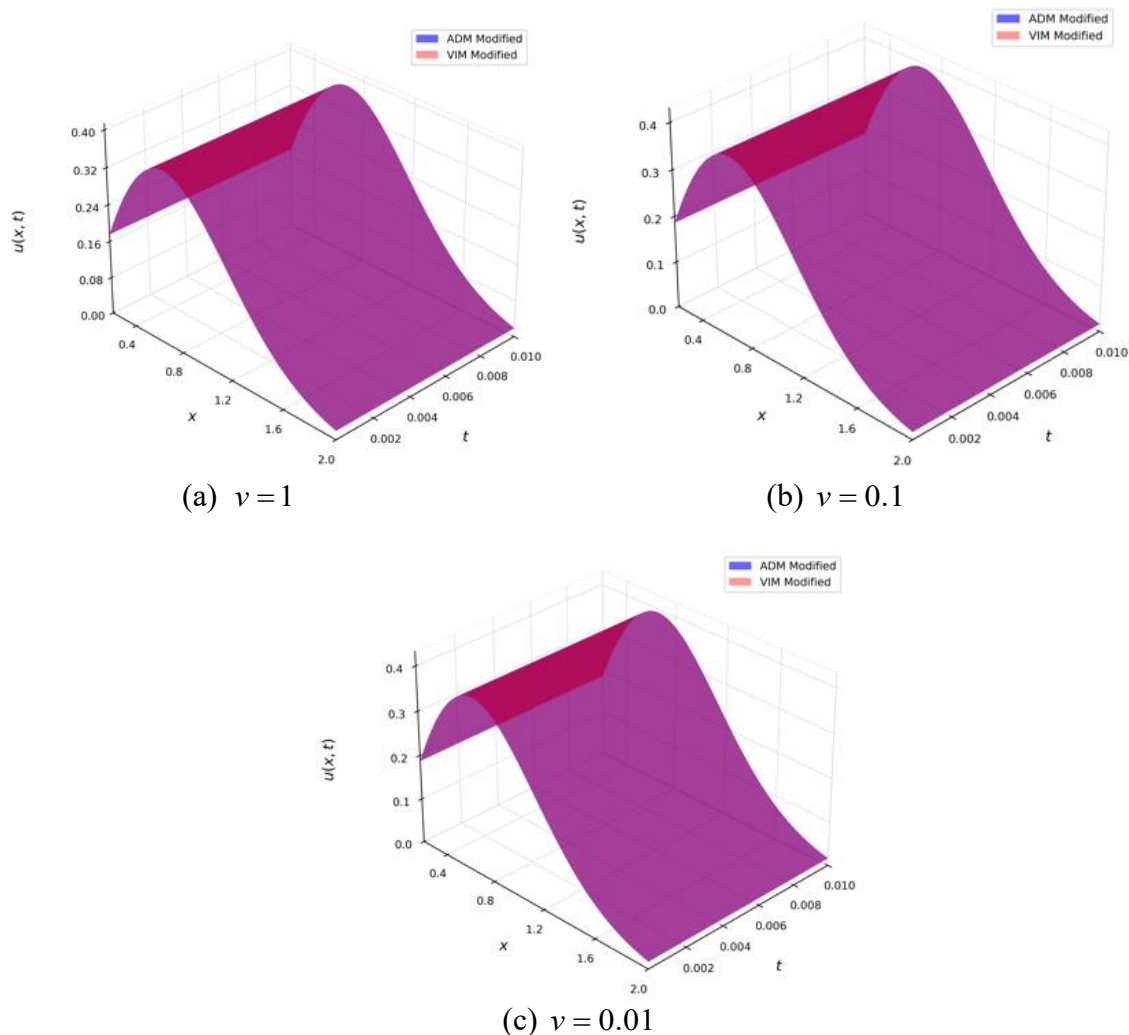


Figure 3: Solution curves of ADM and VIM for different values of ν for Problem 3

Figures 1, 2 and 3 give the solution curves approached by ADM and VIM for various values of ν . As the value of ν increases, there is no visually discernible change in the overall shape of the graphs. This shows that the inclusion of the damping term does not adversely affect the stability characteristics of either technique, as demonstrated by the complete overlap of the results obtained by ADM and VIM. This, therefore, implies that the inclusion of this term is inconsequential to the relative performances of ADM and VIM, as both techniques are equivalent in their stability characteristics.

Conclusion

This study successfully confirming the effectiveness of both ADM and VIM in solving Burgers' equation. Although both approaches produce nearly identical numerical results, the VIM may be regarded as computationally more elegant due to the incorporation of the Lagrange multiplier, which removes the mathematically intensive procedure required for constructing Adomian polynomials in the ADM. This is proven by the execution time data from the Maple computations, which show that the VIM execute time is shorter than that of ADM. Finally future research should extend the comparative analysis to multidimensional Burger's equations to assess the scalability and efficiency of both methods as spatial complexity increases. Furthermore, long term performance analysis is necessary to evaluate the stability of these methods during shock formation and propagation.

Acknowledgements

The authors are highly grateful to Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA (UiTM) Cawangan Kelantan for conducive research environment provided.

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