

ANALYZING ONLINE ACADEMIC INTERACTIONS THROUGH FUZZY AND NEUTROSOPHIC RELATIONAL MAPS

Norarida Abd Rhani ^{1*}
Nur Ariana Mohd Arman ²
Tihani Batrisyia Abd Rashid ³
Siti Nurul Fitriah Mohamad ¹
Hazwani Hashim ¹
Firdawati Mohamed ¹

¹ Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Cawangan Kelantan, 18500 Machang, Kelantan (E-mail: norarida@uitm.edu.my, fitriah@uitm.edu.my, hazwanishashim@uitm.edu.my, firdawati02@uitm.edu.my)

² Farmasi LAUTAN Tingkat Bawah, Lot 61187, Taman D'Harah Kampung Raja, 22200, Besut, Terengganu (E-mail: ariananur904@gmail.com)

³ HOME Maths Therapy HQ 68 & 69, Jln Meru Bestari B7, Medan Meru Bestari, 30020 Ipoh, Perak, Malaysia (E-mail: t.batrisyia.rsh@gmail.com,)

*Corresponding author: norarida@uitm.edu.my

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Abstract: *The transition to Online Distance Learning (ODL) during the COVID-19 pandemic has changed lecturer–student interactions and created challenges in maintaining students' academic performance. This study applies Fuzzy Relational Maps (FRM) and Neutrosophic Relational Maps (NRM) to analyse the causal relationships between lecturers' teaching behaviours and students' academic performance in an ODL environment. Expert opinions from a lecturer and a student at Universiti Teknologi MARA (UiTM) Cawangan Kelantan are used to construct relational matrices and directed graphs representing teaching behaviours and academic performance. Iterative matrix operations were performed to identify fixed points that represent stable interaction patterns. The findings show that FRM provides clearer representations of causal relationships, while NRM is more effective in representing relationships involving uncertainty and indeterminacy. The study demonstrates that FRM and NRM can be useful tools for modelling lecturer–student interactions and supporting educational decision-making. The findings also provide a basis for future research on academic performance under uncertain learning conditions.*

Keywords: *Online Distance Learning; COVID-19 pandemic; Fuzzy Relational Maps; Neutrosophic Relational Maps*

Introduction

The sudden transition to Online Distance Learning (ODL) triggered by the coronavirus disease (COVID-19) pandemic has significantly changed the sector of higher education. As noted in Mathew & Chung (2020), COVID-19 first discovered in Wuhan, China, in December 2019, spread quickly around the world and killed a significant number of people. As a result, the use of digital technology in education at all learning stages has grown in significance and influences the practices of educational institutions, teachers, and students.

Both students and lecturers faced new difficulties in sustaining effective communication, engagement, and academic performance as in person instruction gave way to remote learning. In this context, understanding the dynamic relationship between lecturers' teaching behaviors and student outcomes has become increasingly critical. Distance learning has existed since the 1850s, when the term "distance education" was first used to describe educational delivery through mediums such as radio, letters, mail, and eventually television (Korkmaz & Toraman, 2020).

The transition from traditional classroom learning to ODL using e-learning courses in Physics is studied by Laumann et al. (2025). The authors employed linear mixed-effects models to analyze the influence of time and setting (before vs. during the COVID-19 pandemic) on students' behavioral engagement, affective engagement, and physics achievement. The findings underscored the importance of engaging students in the learning process, particularly during extended periods of distance learning, and highlighted the challenges faced in Physics instruction, especially in relation to practical work.

A quantitative analysis, including calculations of mean, median, and frequency, were conducted by Nik Fauzi et al. (2025). The authors investigated Muslim students' experiences with post-pandemic learning, with an emphasis on Islamic elements. The findings revealed that most Muslim students perceived post-pandemic learning positively, aligning with Islam's emphasis on the pursuit of knowledge. However, there are limitations to the switch to ODL. This includes problems in managing time, less motivation, and social isolation.

The complex and uncertain nature of human behavior in digital learning platforms makes it challenging for traditional statistical methods like correlation and regression analysis to effectively capture. To overcome these limitations, Fuzzy Relational Maps (FRM) and Neutrosophic Relational Maps (NRM) were used in this study. FRM and NRM provide flexible modelling and suitable for representing uncertain causal relationships. In 1965, Zadeh presented fuzzy set theory as a solution to uncertainty related issues. This work was later continued in 1977 by Dubois & Prade (1978), who used the fuzzy concept to expand the idea of fuzzy functions.

In educational fields where subjective assessments dominate, FRM and NRM are particularly helpful. Unlike traditional soft computing techniques like neural networks or probabilistic reasoning, which require big data and numerical inputs, FRM and NRM enable modelling based on conceptual relationships and expert opinions. FRM better at representing the distinction between the teaching behaviors of lecturers and the academic performance of their students. NRM includes indeterminacy, enabling the model to represent ambiguity and incomplete information frequently found in real academic settings.

Fuzzy Relational Maps, initially introduced by Vasantha & Sultana (2000), are a specialized form of Fuzzy Cognitive Maps (FCM). In 1986, Kosko originally put on the idea of FCM. The way that FCM and FRM illustrate causal relationships is where they diverge most. FRM organizes these relationships into two separate sets: a domain and a range space, whereas FCM emphasizes relationships between simultaneously active components. This separation is especially advantageous when analyzing interactions between two different entities, such as teacher-student, employer-employee, or doctor-patient relationships. More recently, Abd Rhani et al. (2024) examined the emergence of COVID-19 during the post-vaccination phase using FCM. Their findings offer valuable insights into the factors contributing to the continued spread of the virus after vaccination and provide guidance on preventive measures to mitigate infection.

FRM demonstrated their effectiveness in decision-making processes, particularly within the medical domain. In the study "The Living Experience of a Diabetic Adult in India Using FRM," for instance, Devadoss & Mystica (2011) used FRM to investigate the connection between risk factors and diabetes symptoms, incorporating opinions from two experts. In the same way, FRM is utilized in Shanmugapriya et al. (2019) to model lifestyle-related diseases within adults in Tamil Nadu mathematically. The authors effectively captured the uncertainty and ambiguity that frequently occur in such medical conditions by examining complex connections between variables such as diet, stress, and other behaviors.

The relationship between risk factors and signs of hypertension was then determined using the FRM approach (Divya & Uduman, 2013). The study proposed preventive measures and identified the main risk factors for hypertension. The authors concluded that one of the main risk factors for cardiovascular diseases is hypertension, which is also referred to as a silent killer. In 2024, Saraswathi et al. (2024) used FRM to investigate several social issues in a particular Chennai community. In the study, the authors set up causes and effects into domain and range spaces within the FRM framework to analyze how they related.

A method for modelling causal relationships between domain and range concepts is provided by FRM. The principles of neutrosophic, degrees of truth (T), falsehood (F), and indeterminacy (I) are added to this concept by NRM, which makes them suitable for investigating complex and vague interactions, like those that occur within educational environments (Kandasamy & Smarandache, 2004). The division of nodes into the domain and range spaces, which each represent different concept groups, is an important attribute of NRM (Devadoss & Ismail, 2013). The concepts are represented as nodes in this framework, while the directional connections between them are illustrated using directed edges (Bivin et al. 2011).

According to Kandasamy & Smarandache (2003), the number of nodes in the domain and range spaces may differ, and nodes that are in between two distinct spaces are not allowed. The suitability of NRM in modelling systems with uncertain information is further highlighted by the possibility that some relationships will remain undecidable. With a focus on the Indian context, the use of NRM has also been shown in analyzing the connection between institutional collaboration aspect and their impact on cross-cultural education (Martin et al. 2024). In the study, these associations are represented by models using expert input from the education sector, enabling neutrosophic to effectively represent uncertainty and indeterminacy.

The current study models how particular lecturer behaviors, such as clarity of instruction, engagement, and feedback, affect students' academic performance using the expert knowledge

of a lecturer and a student at Universiti Teknologi MARA (UiTM) Cawangan Kelantan. Two main scenarios are focused on the methodology: (1) the lecturer provides clear instructions, and (2) the lecturer provides unclear instructions. These cases are selected in accordance with the experts' complete responses.

By comparing the output patterns produced by both FRM and NRM analyses, this research aims to deepen understanding of how teaching strategies affect student performance in ODL environments. The findings can support the development of more effective pedagogical strategies and institutional policies, especially in contexts where remote or hybrid education remains prevalent.

Preliminaries

In this section, some existing definitions which are already defined by many researchers are discussed. The following are some basic concepts and definitions related to this study.

Definition 1 (Kandasamy & Smarandache, 2003) Fuzzy Relational Maps (FRM) is a directed graph or map from D to R that reflects the causal connections spaces D and R by having notions like policies or occurrences as nodes and causalities as edges. Denote the two nodes of a FRM as D_i and R_j . The relational causation of D_i on R_j is indicated by the directed edge from D_i to R_j and every edges in FRM is given a weighted value in the set $\{0,1,-1\}$. Assume e_{ij} is the weight of the edge $D_i R_j, (e_{ij}) \in \{0,1,-1\}$.

Definition 2 (Kandasamy & Smarandache, 2003) Neutrosophic Relational Maps (NRM) is a directed graph or map from D to R that reflects the causal connections spaces D and R by having notions like policies or occurrences as nodes and causalities as edges. Let D be the domain space and R be the range space, with $D_1, \dots, D_n$ representing the conceptual nodes of the domain space D and $R_1, \dots, R_m$ where $m, n \in \mathbb{N}$ representing the conceptual nodes of the range space R , respectively. This will result in the disjoint class $D \cap R = \emptyset$. Assume that a FRM exists between D and R and that at least one edge between $D_i R_j$ is ambiguous.

Definition 3 (Kandasamy & Smarandache, 2003) Simple Fuzzy Relational Maps (FRM) is FRM having edge weights of 0 and 1 and simple Neutrosophic Relational Maps (NRM) is take edge values from 0, 1, -1 and I .

Definition 4 (Kandasamy & Smarandache, 2003) The matrix E is referred to as the relational matrix of the Fuzzy Relational Maps (FRM) where the definition is given as $E = (e_{ij})$, where (e_{ij}) is the weight of the directed edge $D_i R_j$ or $R_j D_i$ and $(e_{ij}) \in \{0,1,-1\}$ Neutrosophic Relational Matrix of NRM is called $N(E)$ where its definition is given as $(e_{ij}) \in \{0,1,-1,I\}$.

Definition 5 (Kandasamy & Smarandache, 2003) The instantaneous state vector of the domain space, or A , represents the nodes' on-off positions at any given moment. Similar to this, let $B = (b_1, \dots, b_m)$ with $b_i \in \{0,1,-1\}$. The on-off position of the nodes at any time is indicated by B , which is referred to as the instantaneous state vector of the range space. If a_i is on for $i=1,2,\dots,n$, $a_i=1$ and $a_i=0$ if a_i is off. For $i=1,2,\dots,m$, $b_i=1$ if b_i is on and $b_i=0$ if b_i is off.

Definition 6 (Kandasamy & Smarandache, 2003) Let $1 \leq j \leq m$, $1 \leq i \leq n$, and $D_i R_j$ or $(R_j D_i)$ be the variables. The dynamical system is said to cycle when R_i (or D_j) is turned on, causality passes via the cycle's edges, and then R_i (or D_j) is caused once more. Any node R_i (or D_j) for $1 \leq i \leq n$, ($1 \leq j \leq m$) holds true for this. This is called as hidden pattern.

Definition 7 (Kandasamy & Smarandache, 2003) Fuzzy Relational Maps (FRM) and Neutrosophic Relational Maps (NRM) with nodes $R_1, R_2, \dots, R_m$ and $D_1, D_2, \dots, D_n$ are considered. Let's set on R_1 (or D_1) as an example to start the dynamical system. In this case, the state vector is $(1, 0, 0, \dots, 1)$ or $(1, 0, 0, \dots, I)$ in R (or $(1, 0, 0, \dots, 1)$ or $(1, 0, 0, \dots, I)$ in D). This state vector is referred to as the fixed point where $m, n \in \mathbb{N} \dots$

Definition 8 (Kandasamy & Smarandache, 2003) If the Fuzzy Relational Maps (FRM) and Neutrosophic Relational Maps (NRM) settles down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \dots \rightarrow A_i \rightarrow A_1$ (or $B_1 \rightarrow B_2 \rightarrow B_3 \rightarrow \dots \rightarrow B_i \rightarrow B_1$) then this equilibrium is called a limit cycle.

Methodology

This study adopts a qualitative modeling approach using FRM and NRM to analyze the impact of lecturer teaching behavior on student academic performance within an ODL environment. The opinion of two experts, one lecturer and one student from UiTM Cawangan Kelantan are provided, which formed the basis for the relational maps.

Justification of Sample

The selection of one lecturer and one student was purposeful, representing both instructional and experiential perspectives. Though limited in number, this expert driven approach is consistent with other studies utilizing FRM or NRM models, such as the work by Shanmugapriya et al. (2019), which also employed insights from two experts. In such studies, detailed qualitative input is prioritized over large sample sizes to capture the depth of causal relationships. Future research may expand expert participation to enhance the generalizability of findings.

Domain and Range Definitions

Based on the linguistic questionnaire and expert opinions from Kandasamy & Smarandache (2003), the key causal relationships are categorized into two distinct sets: the domain space (Lecturer) and the range space (Student), which are disjoint in terms of their conceptual representations.

The domain space (D) consists of eight elements reflecting the lecturers' teaching behaviors:

- D_1 - Lecturer provides clear instruction
- D_2 - Lecturer provides unclear instruction
- D_3 - Lecturer gives response to student queries
- D_4 - Lecturer does not give response to student queries
- D_5 - Engagement in online discussions
- D_6 - No engagement in online discussions
- D_7 - Lecturer discusses the given exercise

D_8 - Lecturer does not discuss the given exercise

The Range space (R) includes five elements representing academic performance:

R_1 - Very high academic performance

R_2 - High academic performance

R_3 - Moderate academic performance

R_4 - Low academic performance

R_5 - Very low academic performance

Fuzzy Relational Maps (FRM) Modelling Process

Each expert's opinion was used to construct directed graphs that link the domain and range concepts. In FRM, causal relationships are assigned weights from the set

$$e_{ij} \in \{0, 1, -1\},$$

where

- 0 represents no significant relationship,
- 1 represents a positive relationship between domain variable D_i and range variable R_i ,
- -1 represents a negative relationship.

FRM of First Expert's Opinion

The experts' opinions are transformed into FRM matrices. Two key scenarios are modeled for each method:

- Case 1: Lecturer provides clear instruction ($D_1 = \text{on}$),
- Case 2: Lecturer provides unclear instruction ($D_2 = \text{on}$).

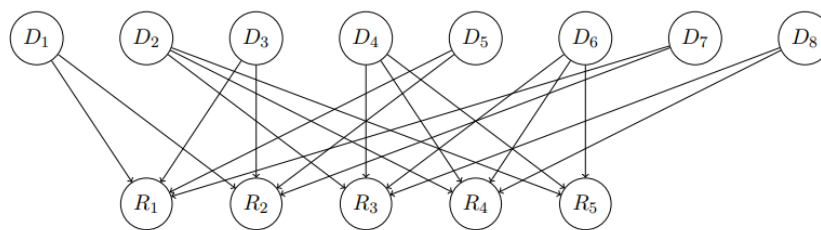


Figure 1: FRM by the First Expert of the Relationship Between Lecturers' Teaching Behaviors and Student's Academic Performance

Matrix A is iteratively processed using matrix vector multiplication to derive the resulting range vectors (student performance) and reveal hidden patterns, such as fixed points or limit cycles, representing equilibrium states.

$$A = \begin{matrix} & \begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$

Case 1: Assume that D_1 is in the “on state” while all other nodes are in the “off state” state, indicating that the lecturers provide clear instructions i.e. $X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$. The hidden pattern of the state vector X_0 is then determined through the following process.

$$X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$$

$$X_0 A^T = (1, 1, 0, 0, 0) = Y_0$$

$$Y_0 A = (2, 0, 2, 0, 2, 0, 2, 0) \rightarrow (1, 0, 1, 0, 1, 0, 1, 0) = X_1$$

$$X_1 A^T = (4, 4, 1, 0, 0) \rightarrow (1, 1, 1, 0, 0) = Y_1$$

$$Y_1 A = (2, 1, 2, 1, 2, 1, 2, 1) \rightarrow (1, 1, 1, 1, 1, 1, 1, 1) = X_2$$

$$X_2 A^T = (8, 8, 4, 4, 3) \rightarrow (1, 1, 1, 1, 1) = Y_2$$

$$Y_2 A = (2, 3, 2, 3, 2, 3, 2, 2) \rightarrow (1, 1, 1, 1, 1, 1, 1, 1) = X_3$$

Therefore, the fixed point obtained is $X_3 = X_2$.

$$X_3 A^T = (4, 4, 4, 4, 3) \rightarrow (1, 1, 1, 1, 1) = Y_3$$

Therefore, the fixed point obtained is $Y_3 = Y_2$.

The results obtained after updating and preserving the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_2 \rightarrow Y_2 \rightarrow X_2 \rightarrow Y_2.$$

Therefore, the fixed points are $X_3 = X_2$ and $Y_3 = Y_2$.

Case 2: Suppose the instantaneous state vector $X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$ the lecturers provide an unclear instruction as D_2 , to be in the “on state”. Now the effect of X_0 on A is as follows:

$$X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$$

$$X_0 A^T = (0, 0, 1, 1, 1) = Y_0$$

$$Y_0 A = (0, 3, 0, 3, 0, 3, 0, 2) \rightarrow (0, 1, 0, 1, 0, 1, 0, 1) = X_1$$

$$X_1 A^T = (0, 0, 4, 4, 3) \rightarrow (0, 1, 1, 1, 1) = Y_1$$

$$Y_1 A = (1, 3, 1, 3, 1, 3, 1, 2) \rightarrow (1, 1, 1, 1, 1, 1, 1, 1) = X_2$$

$$X_2 A^T = (4, 4, 4, 4, 3) \rightarrow (1, 1, 1, 1, 1) = Y_2$$

$$Y_2 A = (2, 3, 2, 3, 2, 3, 2, 2) \rightarrow (1, 1, 1, 1, 1, 1, 1, 1) = X_3$$

Therefore, the fixed point obtained is $X_3 = X_2$.

$$X_3 A^T = (4, 4, 4, 4, 3) \rightarrow (1, 1, 1, 1, 1) = Y_3$$

Therefore, the fixed point obtained is $Y_3 = Y_2$.

The results obtained after updating and preserving the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_2 \rightarrow Y_2 \rightarrow X_2 \rightarrow Y_2.$$

Therefore, the fixed points are $X_3 = X_2$ and $Y_3 = Y_2$.

FRM of Second Expert's Opinion

The opinion of the second expert is formulated based on the responses received, and the corresponding directed graph is presented below.

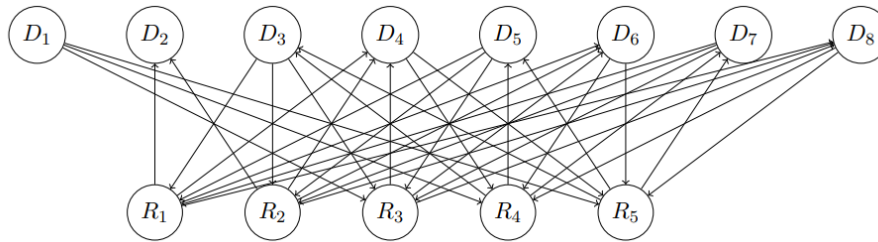


Figure 2: FRM by the Second Expert of the Relationship Between Lecturers' Teaching Behaviors and Student's Academic Performance

This opinion is converted into the fuzzy relational matrix B , as shown below:

$$B = \begin{matrix} & \begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{matrix} & \begin{pmatrix} 0 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 0 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \end{matrix}$$

Case 1: Now, the state D_1 as "on state" ('The lecturers provide clear instruction') while all other nodes are in the "off state" state in the domain space. The following process is to obtain the hidden pattern of the state vector $X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$.

$$X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$$

$$X_0 B^T = (0, 0, 1, 1, 1) = Y_0$$

$$Y_0 B = (3, 0, -1, 1, -1, 1, -1, 1) \rightarrow (1, 0, 0, 1, 0, 1, 0, 1) = X_1$$

$$X_1 B^T = (-3, -3, -2, 4, 4) \rightarrow (1, 0, 0, 1, 1) = Y_1$$

$$Y_1 B = (2, -1, -1, 1, -1, 1, -1, 1) \rightarrow (1, 0, 0, 1, 0, 1, 0, 1) = X_2$$

Therefore, the fixed point obtained is $X_2 = X_1$.

$$X_2 B^T = (-3, -3, -2, 4, 4) \rightarrow (1, 0, 0, 1, 1) = Y_2$$

Therefore, the fixed point obtained is $Y_2 = Y_1$.

After updating and preserving the instantaneous vector, the results generated by X_0 is obtained as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_1 \rightarrow Y.$$

Therefore, the fixed points are $X_2 = X_1$ and $Y_2 = Y_1$.

Case 2: The state vector D_2 is “on state” (‘The lecturers provide unclear instruction’) while all other nodes “off state” state in the range space. The hidden pattern of the state vector $X_0 = (0,1,0,0,0,0,0,0)$ is derived by the following process.

$$X_0 = (0,1,0,0,0,0,0,0)$$

$$X_0 B^T = (-1,-1,0,0,0) = Y_0$$

$$Y_0 B = (0,2,-2,,2,-2,2,-2,2) \rightarrow (0,1,0,1,0,1,0,1) = X_1$$

$$X_1 B^T = (-4,-4,-3,3,3) \rightarrow (0,1,0,1,1) = Y_1$$

$$Y_1 B = (2,-1,-1,1,-1,1,-1,1) \rightarrow (1,1,0,1,0,1,0,1) = X_2$$

$$X_2 B^T = (-4,-4,-2,4,4) \rightarrow (0,1,0,1,1) = Y_2$$

Therefore, the fixed point obtained is $Y_2 = Y_1$.

$$Y_2 B = (2,-1,-1,1,-1,1,-1,1) \rightarrow (1,1,0,1,0,1,0,1) = X_3$$

Therefore, the fixed point obtained is $X_3 = X_2$.

The results generated by X_0 after updating and retaining the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_2 \rightarrow Y_1 \rightarrow X_2.$$

Therefore, the fixed points are $Y_2 = Y_1$ and $X_3 = X_2$.

Neutrosophic Relational Maps (NRM) Modelling Process

In NRM, relationships can also include indeterminacy, represented by “ I ” (in addition to 1,0,-1). This is essential for capturing ambiguous influences. In this study, the symbol “ I ” is treated symbolically to retain its qualitative meaning of indeterminacy. During matrix iterations, if an input node with an active state maps to an “ I ” in the relational matrix, the corresponding output node is updated as “ I ”, indicating an undecidable influence. No numeric approximation is applied to “ I ”, in line with neutrosophic modeling standards (Kandasamy & Smarandache, 2003). Causal relationships are assigned weights from the set

$$e_{ij} \in \{0,1,-1,I\},$$

Where

I is an indeterminate relationship.

NRM of First Expert’s Opinion

The experts’ opinions are transformed into NRM matrices. Two key scenarios are modeled for each method:

- Case 1: Lecturer provides clear instruction ($D_1 = \text{on}$),
- Case 2: Lecturer provides unclear instruction ($D_2 = \text{on}$).

The matrices are iteratively processed using matrix vector multiplication to derive the resulting range vectors (student performance) and reveal hidden patterns, such as fixed points or limit cycles, representing equilibrium states. The first expert's opinion is derived based on the responses received. The corresponding directed graph is presented below.

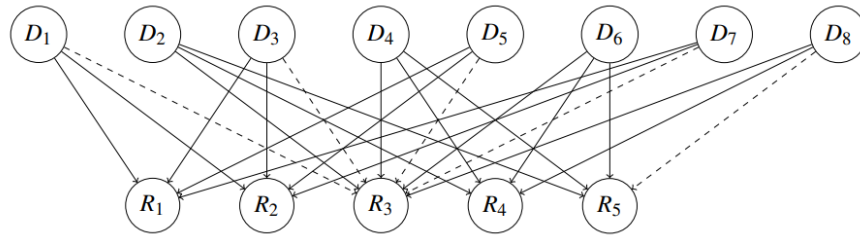


Figure 3: NRM by the First Expert of the Relationship Between Lecturers' Teaching Behaviors and Student's Academic Performance

Figure 3 is then represented in the form of NRM matrix as shown below.

$$N(A) = \begin{matrix} & \begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ I & 1 & I & 1 & I & 1 & I & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & I \end{pmatrix} \end{matrix}$$

Case 1: Let D_1 be kept in "on state", i.e., "The lecturers provide clear instruction" is in the "on state". Let the hidden pattern of the state vector $X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$ be obtained by the following method:

$$X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$$

$$X_0 N(A)^T = (1, 1, I, 0, 0) = Y_0$$

$$Y_0 N(A) = (2 + I^2, I, 2 + I^2, I, 2 + I^2, I) \rightarrow (1, I, 1, I, 1, I) = X_1$$

$$X_1 N(A)^T = (4, 4, 8I, 4I, 3I + I^2) \rightarrow (1, 1, I, I, I) = Y_1$$

$$Y_1 N(A) = (2 + I^2, 2I + 1, 2 + I^2, 2I + 1, 2 + I^2, 3I) \rightarrow (1, I, 1, I, 1, I) = X_2$$

Therefore, the fixed point obtained is $X_2 = X_1$.

$$X_2 N(A)^T = (4, 4, 8I, 4I, 3I + I^2) \rightarrow (1, 1, I, I, I) = Y_2$$

Therefore, the fixed point obtained is $Y_2 = Y_1$.

The results after updating and retaining the instantaneous vector are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_1 \rightarrow Y_1.$$

Therefore, the fixed points are $X_2 = X_1$ and $Y_2 = Y_1$.

Case 2: The following process is to obtain the hidden pattern of the state $X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$.

$$X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$$

$$X_0 N(A)^T = (0, 0, 1, 1, 1) = Y_0$$

$$Y_0 N(A) = (I, 3, I, 3, I, 3, I, 2 + I) \rightarrow (I, 1, I, 1, I, 1, I, 1) = X_1$$

$$X_1 N(A)^T = (4I, 4I, 4I^2 + 4, 4, 3 + I) \rightarrow (I, I, 1, 1, 1) = Y_1$$

$$Y_1 N(A) = (3I, 3, 3I, 3, 3I, 3, 3I, 2 + I) \rightarrow (I, 1, I, 1, I, 1, I, 1) = X_2$$

Therefore, the fixed point obtained is $X_2 = X_1$.

$$X_2 N(A)^T = (4I, 4I, 4I^2 + 4, 4, 3 + I) \rightarrow (I, I, 1, 1, 1) = Y_2$$

Therefore, the fixed point obtained is $Y_2 = Y_1$.

The results generated by X_0 after updating and retaining the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_1 \rightarrow Y_1.$$

Therefore, the fixed points are $X_2 = X_1$ and $Y_2 = Y_1$.

NRM of Second Expert's Opinion

Based on the responses received, the opinion of the second expert is derived. The corresponding directed graph is presented below.

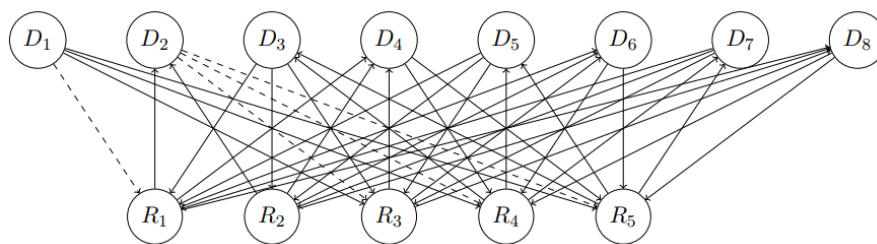


Figure 4: NRM by the Second Expert of the Relationship Between Lecturers' Teaching Behaviors and Student's Academic Performance

This opinion is represented in the form of a neutrosophic relational matrix, denoted as $N(B)$, as shown below:

$$N(B) = \begin{matrix} & \begin{matrix} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{matrix} & \begin{pmatrix} I & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & I & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & I & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & I & -1 & 1 & -1 & 1 & -1 & 1 \end{pmatrix} \end{matrix}$$

Case 1: Now, the state as “on state” (‘The lecturers provide clear instruction’) while all other nodes are in the “off state” state in domain space. The hidden pattern of the state vector is obtained as follows:

$$X_0 = (1, 0, 0, 0, 0, 0, 0, 0)$$

$$X_0 N(B)^T = (I, 0, 1, 1, 1) = Y_0$$

$$Y_0 N(B) = (I^2 + 3, 2I, I - 1, -I + 1, I - 1, -I + 1, I - 1, -I + 1) \rightarrow (1, I, 0, 0, 0, 0, 0, 0) = X_1$$

$$X_1 N(B)^T = (0, -I, 1 + I^2, 1 + I^2, 1 + I^2) \rightarrow (0, 0, 1, 1, 1) = Y_1$$

$$Y_1 N(B) = (3, 3I, -1, 1, -1, 1, -1, 1) \rightarrow (1, I, 0, 1, 0, 1, 0, 1) = X_2$$

$$X_2 N(B)^T = (-3, -I - 3, I^2 - 2, I^2 + 4, I^2 + 4) \rightarrow (0, 0, 0, 1, 1) = Y_2$$

$$Y_2 N(B) = (2, 2I, -2, 2, -2, 2, -2, 2) \rightarrow (1, I, 0, 1, 0, 1, 0, 1) = X_3$$

Therefore, the fixed point obtained is $X_3 = X_2$.

$$X_3 N(B)^T = (-3, -I - 3, I^2 - 2, I^2 + 4, I^2 + 4) \rightarrow (0, 0, 0, 1, 1) = Y_3$$

Therefore, the fixed point obtained is $Y_3 = Y_2$.

The results generated by X_0 after updating and retaining the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_2 \rightarrow Y_2 \rightarrow X_2 \rightarrow Y_2.$$

Therefore, the fixed points are $X_3 = X_2$ and $Y_3 = Y_2$.

Case 2: The idea of D_2 , which is lecturer provides unclear instruction. Take $X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$ as a state vector. The impact of X_0 on $N(B)$ is given by:

$$X_0 = (0, 1, 0, 0, 0, 0, 0, 0)$$

$$X_0 N(B)^T = (-1, -1, I, I, I) = Y_0$$

$$Y_0 N(B) = (2I, 2 + 3I^2, -2 - I, 2 + I, -2 - I, 2 + I, -2 - I, 2 + I) \rightarrow (I, 1, 0, 1, 0, 1, 0, 1) = X_1$$

$$X_1 N(B)^T = (I^2 - 4, -4, 2I - 3, 2I + 3, 2I + 3) \rightarrow (0, 0, 0, 1, 1) = Y_1$$

$$Y_1 N(B) = (2I, 2I, -2, 2 - 2, 2, -2, 2) \rightarrow (I, 1, 0, 1, 0, 1, 0, 1) = X_2$$

Therefore, the fixed point obtained is $X_2 = X_1$.

$$X_2 N(B)^T = (I - 4, -4, I - 2, I + 4, I + 4) \rightarrow (0, 0, 0, 1, 1) = Y_2$$

Therefore, the fixed point obtained is $Y_2 = Y_1$.

The results generated by X_0 after updating and retaining the instantaneous vector at each step are as follows:

$$X_0 \rightarrow Y_0 \rightarrow X_1 \rightarrow Y_1 \rightarrow X_1 \rightarrow Y_1.$$

Therefore, the fixed points are $X_2 = X_1$ and $Y_2 = Y_1$.

Results and Discussion

The results from both FRM and NRM highlight different aspects of the teaching-learning dynamic under ODL settings. Tables 1-3 present the set of relational matrices of FRM and NRM illustrating the relationship between lecturers' teaching behaviors and students' academic performance.

Table 1: FRM and NRM Between Two Experts Based on D_1 and D_2

Expert	FRM $\{(X),(Y)\}$	NRM $\{(X),(Y)\}$
1	$D_1 = \{(1,1,1,1,1,1,1,1), (1,1,1,1,1,1)\}$ $D_2 = \{(1,1,1,1,1,1,1,1), (1,1,1,1,1,1)\}$	$D_1 = \{(1,I,1,I,1,I,1,I), (1,1,I,I,I,I)\}$ $D_2 = \{(I,1,I,1,I,1,I,1), (I,I,1,1,I,I)\}$
2	$D_1 = \{(1,0,0,1,0,1,0,1), (1,0,0,1,1)\}$ $D_2 = \{(1,1,0,1,0,1,0,1), (0,1,0,1,1)\}$	$D_1 = \{(1,I,0,1,0,1,0,1), (0,0,0,1,1)\}$ $D_2 = \{(I,1,0,1,0,1,0,1), (0,0,0,1,1)\}$

In Case 1 (clear instruction), both models identify consistent influence patterns between the domain and range spaces, with FRM clearly mapping behaviors like responsiveness and discussion engagement to high academic outcomes. NRM further reveals indeterminacies in lower performance outcomes, suggesting some effects may not be strictly deterministic.

In Case 2 (unclear instruction), FRM reveals structured patterns, although the relationships appear more variable compared to Case 1. NRM, however, introduces significant indeterminacy, indicating that student performance in such cases is subject to variable or context dependent interpretations.

FRM offers a structured and straightforward approach by presenting binary cause effect relationships, making it especially useful for identifying dominant teaching behaviors that influence student performance. In contrast, NRM extends this capability by incorporating indeterminate relationships, allowing it to model real world complexity and ambiguity more effectively. This enables NRM to capture significant patterns that FRM alone may overlook, making it particularly valuable in contexts where uncertainty and incomplete information are present.

Lecturers may apply findings from the FRM to enhance their teaching strategies by focusing on clearer instruction and increased responsiveness, ultimately enhancing student academic performance. In contrast, insights gained from NRM emphasize the need to consider contextual uncertainties such as varying levels of student motivation, unequal access to learning resources, and external stress factors which play a significant role in shaping learning outcomes within more complex and unpredictable educational environments.

These findings reinforce the value of combining FRM and NRM in educational diagnostics and decision making, offering both clarity and depth for strategic planning in ODL context.

Conclusion

This study showed how effectively FRM and NRM model complex relationships between lecturers' teaching behaviors and students' academic performance in ODL context. While NRM reveals hidden indeterminacies that provide insights into adaptive strategies, FRM efficiently maps direct causal relationships. This study used one lecturer and one student who were selected based on their experience in online academic interactions. Since the main objective was to demonstrate the application of Fuzzy Relational Maps (FRM) and Neutrosophic Relational Maps (NRM), expert knowledge was prioritized over a large sample size. However, the authors acknowledge that using only two experts may introduce bias and limit the generalizability of the findings. Future studies should include more lecturers and students to obtain a consensus-based relational matrix and improve the reliability of the results.

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